UNSTABLE ROTATIONAL STATES OF CLOSED STRING WITH MASSIVE POINTS

G. S. Sharov

Tver state university

Tver, 170002, Sadovyj per. 35, Mathem. dep-t

Abstract

For the closed string carrying 2 or 3 point-like masses the stability problem for central and linear rotational states is considered. This problem is important for applications of these model to describing baryons, glueballs or other exotic hadrons. The linear rotational state correspond to an uniform rotation of the system with rectilinear string segments, connecting massive points. The state is named "central" one, if there is a massive point at the rotational center.

It is shown that the linear rotational states with 2 massive points are stable with respect to small disturbances. But the central rotational states with 3 masses are not stable, if the central mass it less than energy of the string with other massive points. This effect may change properties of excited hadron states, in particular, increase their width.

1. Introduction

Various string models of mesons, baryons and exotic hadrons include a system of massive points connected by strings [1] – [11]. In these models the massive points describe quarks or constituent gluons and the Nambu-Goto string simulates strong interaction between them and describes the QCD confinement mechanism.

The mentioned models includes the string with massive ends as the meson string model q-$\overline{q}$ [1] or the quark-diquark baryon model q-qq [5]; the linear string baryon model q-q-q [3, 4]; the Y baryon model ${}^{q}\mathsf{Y}^{q}_{q}$ [2, 6]; the "triangle" (or Δ) baryon configuration ${}_{q}\overset{q}{\triangle}_{q}$ [7], and (generalizing the last model) the closed string with n massive ends [10, 11].

If we use a set of rotational states (planar uniform rotations of a system), all the listed string hadron models generate linear or quasilinear Regge trajectories $J \simeq \alpha_0 + \alpha' E^2$, and may be applied for describing excited hadron states with high angular momenta J are energies (masses) E [4] – [11].

In these applications the problem of stability for rotational states with respect to small disturbances is very important for choosing the most adequate string model for baryons [4], for glueballs and other exotic hadrons [10] – [17]. Another reason for interest to this stability problem is the existence of quasirotational states [18, 19] in the linear vicinity of *stable* rotational states describing radial excitations or daughter Regge trajectories for hadrons.

The stability problem for rotational states is solved for the string with massive ends [18, 19], for the linear string baryon model q-q-q [20]; and for the Y baryon configuration [21]. Analytical investigations and numerical simulations of small disturbances demonstrated that rotational

states of the string with massive ends are stable [18, 19], but they are unstable for string baryon models q-q-q and Y [20, 21]. For the last two models there are exponentially growing modes in spectra of small disturbances.

In this paper the stability problem is solved for the certain class of rotational states of the closed string carrying n point-like masses. We consider the case $n = 3$ and the rotational states (named central states) with a massive point at the rotational center. In the particular case, if the central mass equals zero, this state is named linear rotational state with $n = 2$. The considered model describes baryons [7] or glueballs [10, 11].

Note that previously the stability problem for rotational states was solved for the closed string with $n = 1$ massive point [22].

2. Dynamics and central rotational states

The classical dynamical equations for the closed string with tension γ carrying n point-like masses m_1, m_2, $\dots$ m_n result from the action [7, 10]

$$A = -\gamma \int \sqrt{(\dot X, X')^2 - \dot X^2 X'^2}\, d\tau d\sigma - \sum_{i=1}^{n} m_i \int \sqrt{\dot x_i^2(\tau)}\, d\tau,$$

and have the form

$$\frac{\partial^2 X^\mu}{\partial\tau^2} - \frac{\partial^2 X^\mu}{\partial\sigma^2} = 0 \qquad (1)$$

$$m_j \frac{d}{d\tau}\frac{\dot x_j^\mu(\tau)}{\sqrt{\dot x_j^2(\tau)}} + \gamma\Big[X'^\mu + \dot\sigma_j(\tau)\dot X^\mu\Big]\Big|_{\sigma=\sigma_j-0} - \gamma\Big[X'^\mu + \dot\sigma_j(\tau)\dot X^\mu\Big]\Big|_{\sigma=\sigma_j+0} = 0, \qquad (2)$$

$$m_n \frac{d}{d\tau}\frac{\dot x_0^\mu(\tau)}{\sqrt{\dot x_0^2(\tau)}} + \gamma\Big[X'^\mu(\tau^*(\tau), 2\pi) - X'^\mu(\tau, 0)\Big] = 0. \qquad (3)$$

if the orthonormality conditions on the world surface $X^\mu(\tau, \sigma)$

$$(\partial_\tau X \pm \partial_\sigma X)^2 = 0, \qquad (4)$$

and the conditions

$$\sigma_0(\tau) = 0, \qquad \sigma_n(\tau) = 2\pi, \qquad (5)$$

are fulfilled [7, 11]. Here $\dot X^\mu \equiv \partial_\tau X^\mu$, $X'^\mu \equiv \partial_\sigma X^\mu$, Scalar product in Minkowski space $R^{1,3}$ is $(a, b) = \eta_{\mu\nu} a^\mu b^\nu$, $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$; speed of light $c = 1$, the parameter σ varies in the limits $\sigma_0 \le \sigma \le \sigma_n$, that is $\sigma \in [0, 2\pi]$, equations $\sigma = \sigma_j(\tau)$ and

$$x^\mu = x_j^\mu(\tau) \equiv X^\mu(\tau, \sigma_j(\tau)), \qquad j = 0, 1, \dots, n$$

determine world lines of the massive points, for the case $j = 0$ and $j = n$ they describe the same trajectory of the n-th point, and their equality forms the closure condition

$$X^\mu(\tau^*, 2\pi) = X^\mu(\tau, 0) \qquad (6)$$

on the tube-like world surface [7, 22]. These equations may contain two different parameters τ and τ^*, connected via the relation $\tau^* = \tau^*(\tau)$. This relation should be included in the closure condition (6) of the world surface.

Conditions (4), (5) always may be fixed without loss of generality, if we choose the relevant coordinates τ, σ [7].

Eqs. (2), (3) are equations of motion for the massive points resulting from the action. They may be interpreted as boundary conditions for Eq. (1).

The system of equations (1) – (6) describe dynamics of the closed string carrying n point-like masses without loss of generality. One also should add that a tube-like world surface of the closed string is continuous one, but its derivatives may have discontinuities at world lines of the massive points (except for derivatives along these lines) [7]. These discontinuities are taken into account in Eqs. (2), (3).

Exact solutions of the system (1) – (6) describing rotational states of the closed string with masses we obtained and classified in Refs. [10, 11]. These states are divided into three classes: hypocycloidal, linear and central rotational states. In hypocycloidal states the rotating string is composed of segments of a hypocycloid. For linear states these segments are rectilinear and all masses m_j move at nonzero velocities v_j. Central states are states with a massive point (or some of them) placed at the rotational center.

The string world surface for the central or linear states can be presented as [10]

$$X^\mu(\tau,\sigma) = x_0^\mu + e_0^\mu a_0\tau + u(\sigma)\cdot e^\mu(\omega\tau). \tag{7}$$

Here

$$e^\mu(\omega\tau) = e_1^\mu\cos\omega\tau + e_2^\mu\sin\omega\tau, \tag{8}$$

the unit vectors

$$e_0^\mu,\ e_1^\mu,\ e_2^\mu,\ e_3^\mu,$$

associated with coordinates x^μ, form the orthonormal basis in $R^{1,3}$. In the closure condition (6) $\tau^* = \tau$. For the solution (7) of Eq. (1) the following values are the constants:

$$\sigma_j(\tau) = \sigma_j = \text{const}, \qquad j = 1,2, \tag{9}$$

$$Q_j = \frac{\gamma}{m_j}\sqrt{\dot{x}_j^2(\tau)}, \qquad j = 1,2,3. \tag{10}$$

The function

$$u(\sigma) = \frac{a_0}{\omega}\cdot\begin{cases} \sin\omega\sigma, & \sigma\in[0,\sigma_1],\\ 2\breve{s}_1\breve{c}_1\cos\omega\sigma + (\breve{s}_1^2-\breve{c}_1^2)\sin\omega\sigma, & \sigma\in[\sigma_1,\sigma_2],\\ -\breve{S}\cos\omega\sigma + \breve{C}\sin\omega\sigma, & \sigma\in[\sigma_2,2\pi] \end{cases} \tag{11}$$

is determined from Eq. (1), continuity of the function $X^\mu(\tau,\sigma)$ on the lines $\sigma = \sigma_j$ and conditions (2) – (6) [10]. Here and below we use the following notations for constants:

$$\breve{c}_1 = \cos\omega\sigma_1, \qquad \breve{s}_1 = \sin\omega\sigma_1, \qquad \breve{C} = \cos 2\pi\omega, \qquad \breve{S} = \sin 2\pi\omega,$$
$$\breve{c}_3 = \cos\omega(2\pi-\sigma_2), \qquad \breve{s}_3 = \sin\omega(2\pi-\sigma_2), \qquad \breve{C}_2 = \cos\omega\sigma_2, \qquad \breve{S}_2 = \sin\omega\sigma_2.$$

Expression (7) describes a rotation of two coinciding string segments, connecting two points with masses m_1, m_2. These points move along circles at velocities v_1 and v_2, the system rotates in the plane e_1, e_2 at angular velocity $\Omega = \omega/a_0$. The point with mass m_3 is at rest at the rotational center ($v_3 = 0$). Values of parameters σ_j, Q_j, m_j, v_j, a_0 are related by the following equations, resulting from Eqs. (2) – (6) [10]:

$$\sigma_2 - \sigma_1 = \pi, \tag{12}$$

$$v_1 = \breve{s}_1 = \sin\omega\sigma_1, \qquad v_2 = \breve{s}_3 = \sin\omega(\pi-\sigma_1), \tag{13}$$

$$2\frac{Q_1}{\omega}=\frac{\breve{s}_1}{\breve{c}_1}=\frac{v_1}{\sqrt{1-v_1^2}},\qquad 2\frac{Q_2}{\omega}=\frac{\breve{s}_3}{\breve{c}_3}=\frac{v_2}{\sqrt{1-v_2^2}}, \tag{14}$$

$$a_0=\frac{m_1Q_1}{\gamma\sqrt{1-v_1^2}}=\frac{m_2Q_2}{\gamma\sqrt{1-v_2^2}}=\frac{m_3Q_3}{\gamma}, \tag{15}$$

$$\frac{m_1v_1}{1-v_1^2}=\frac{m_2v_2}{1-v_2^2}. \tag{16}$$

If the values v_1 and v_2 are given, one can calculate from Eqs. (13) the following values:

$$\omega=\frac{(-1)^{k_1}\arcsin v_1+(-1)^{k_2}\arcsin v_2}{\pi}+2(k_1+k_2),\quad \sigma_1=\frac{(-1)^{k_1}\arcsin v_1+2\pi k_1}{\omega}. \tag{17}$$

Here k_1 and k_2 are arbitrary integers resulting in the inequalities $0<\sigma_1<\pi$, $\breve{s}_1>0$, $\breve{s}_3>0$. The simplest case $k_1=k_2=0$ corresponds to 2 coinciding rectilinear string segments, connecting the points m_1, m_2. For other permissible values k_1 and k_2 these segments are kinked curves, folded on one (rotating) straight line. The fold points move along circles at the speed of light.

3. Stability problem for central rotational states

Possible applications of solutions (7) in hadron spectroscopy essentially depend on stability or instability of these states with respect to small disturbances. In this section we study spectrum of these disturbances for the central rotational states.

This problem has been recently solved for the closed string with $n=1$ massive point in Ref. [**?**]. In Ref. [10] central rotational states with $n=3$ are considered, but some simplifying assumptions in this paper appeared to be incorrect, and the results need refinement.

To solve the stability problem for the central rotational states (7) we consider the general solution of Eq. (1) for the string with 3 masses

$$X^\mu(\tau,\sigma)=\frac{1}{2}[\Psi^\mu_{j+}(\tau+\sigma)+\Psi^\mu_{j-}(\tau-\sigma)],\qquad \sigma\in[\sigma_{j-1},\sigma_j],\quad j=1,2,3. \tag{18}$$

Here the functions $\Psi^\mu_{j\pm}(\tau\pm\sigma)$ are smooth, the world surface (18) is smooth between world lines of massive points.

We denote $\Psi^{(r)\mu}_{j\pm}$ the functions in the expression (18) for the rotational states (7). Their derivatives $\dot\Psi^{(r)\mu}_{j\pm}\equiv\frac{d}{d\tau}\Psi^{(r)\mu}_{j\pm}$ in accordance with Eq. (11) are

$$\begin{aligned}
\dot\Psi^{(r)\mu}_{1\pm}(\tau)&=a_0\Big[e_0^\mu\pm e^\mu(\omega\tau)\Big],\\
\dot\Psi^{(r)\mu}_{2\pm}(\tau)&=a_0\Big[e_0^\mu+2v_1\breve{c}_1\acute{e}^\mu(\omega\tau)\pm(2v_1^2-1)\,e^\mu(\omega\tau)\Big],\\
\dot\Psi^{(r)\mu}_{3\pm}(\tau)&=a_0\Big[e_0^\mu-\breve{S}\acute{e}^\mu(\omega\tau)\pm\breve{C}e^\mu(\omega\tau)\Big],
\end{aligned} \tag{19}$$

Here the rotating vector

$$\acute{e}^\mu(\omega\tau)=-e_1^\mu\sin\omega\tau+e_2^\mu\cos\omega\tau,$$

is orthogonal to the vector $e^\mu(\omega\tau)$ (8).

To describe any small disturbances of the rotational motion, that is motions close to states (7) we consider vector functions $\dot\Psi^\mu_{j\pm}$ close to $\dot\Psi^{(r)\mu}_{j\pm}$ in the form

$$\dot\Psi^\mu_{j\pm}(\tau)=\dot\Psi^{(r)\mu}_{j\pm}(\tau)+\varphi^\mu_{j\pm}(\tau). \tag{20}$$

The disturbance $\varphi^\mu_{j\pm}(\tau)$ is supposed to be small, so we omit squares of $\varphi_{j\pm}$ when we substitute the expression (20) into dynamical equations (2), (3) and (6). In other words, we work in the first linear vicinity of the states (7). Both functions $\Psi^\mu_{j\pm}$ and $\Psi^{(r)\mu}_{j\pm}$ in expression (20) must satisfy the condition

$$\dot\Psi^{\,2}_{j+}=\dot\Psi^{\,2}_{j-}=0,$$

resulting from Eq. (4), hence in the first order approximation in $\varphi_{j\pm}$ the following scalar product equals zero:

$$(\dot\Psi^{(r)}_{j\pm},\varphi_{j\pm})=0. \tag{21}$$

For the disturbed motions the equalities $\tau^*=\tau$ and $\sigma_j(\tau)=\sigma_j=\mathrm{const}$ (9), generally speaking, is not carried out and should be replaced with the equalities

$$\sigma_1(\tau)=\sigma_1+\delta_1(\tau),\qquad \sigma_2(\tau)=\sigma_2+\delta_2(\tau),\qquad \tau^*=\tau+\delta(\tau), \tag{22}$$

where $\delta_j(\tau)$ and $\delta(\tau)$ are small disturbances.

Expression (20) together with Eq. (18) is the solution of the string motion equation (1). Therefore we can obtain equations of evolution for small disturbances $\varphi^\mu_{j\pm}(\tau)$, substituting expressions (20) and (22) with Eq. (19) into other equations of motion (2), (3), the closure condition (6) and the continuity condition

$$X^\mu(\tau,\sigma_j(\tau)-0)=X^\mu(\tau,\sigma_j(\tau)+0),\qquad i=1,2 \tag{23}$$

in linear approximation. We are to take into account nonlinear factors $\Bigl\{\bigl[\frac{d}{d\tau}X(\tau,\sigma_j(\tau))\bigr]^2\Bigr\}^{-1/2}$ and contributions from the disturbed arguments τ^* and $\sigma_j(\tau)$ (22), for example:

$$\dot\Psi^{(r)\mu}_{3\pm}(\tau^*\pm2\pi)\simeq\dot\Psi^{(r)\mu}_{3\pm}(\tau\pm2\pi)+\delta(\tau)\,\ddot\Psi^{(r)\mu}_{3\pm}(\tau\pm2\pi).$$

This substitution for the central rotational state (7) with $n=3$ and vector-functions $\dot\Psi^{(r)\mu}_{j\pm}$ (19) after simplifying results in the following system of 6 vector equations in linear (with respect to $\varphi^\mu_{j\pm}$, δ_j and δ) approximation:

$$\begin{gathered}
\varphi^\mu_{1+}(+_1)+\varphi^\mu_{1-}(-_1)-\varphi^\mu_{2+}(+_1)-\varphi^\mu_{2-}(-_1)+4\breve c_1a_0[e^\mu(\omega\tau)\,\dot\delta_1(\tau)+\omega\acute e^\mu(\omega\tau)\,\delta_1]=0,\\
\varphi^\mu_{2+}(+_2)+\varphi^\mu_{2-}(-_2)-\varphi^\mu_{3+}(+_2)-\varphi^\mu_{3-}(-_2)-4\breve c_3a_0[e^\mu(\omega\tau)\,\dot\delta_2(\tau)+\omega\acute e^\mu(\omega\tau)\,\delta_2]=0,\\
\varphi^\mu_{3+}(+)+\varphi^\mu_{3-}(-)-\varphi^\mu_{1+}(\tau)-\varphi^\mu_{1-}(\tau)+2a_0e^\mu_0\dot\delta(\tau)=0,\\
\tfrac{d}{d\tau}\Bigl\{\varphi^\mu_{1+}(+_1)+\varphi^\mu_{1-}(-_1)+2\breve c_1a_0(e^\mu\dot\delta_1+\omega\acute e^\mu\delta_1)+G_1(e^\mu_0+v_1\acute e^\mu)\Bigr\}+\\
+Q_1\Bigl[\varphi^\mu_{1+}(+_1)-\varphi^\mu_{1-}(-_1)-\varphi^\mu_{2+}(+_1)+\varphi^\mu_{2-}(-_1)\Bigr]=0.\\
\tfrac{d}{d\tau}\Bigl\{\varphi^\mu_{2+}(+_2)+\varphi^\mu_{2-}(-_2)-2\breve c_3a_0(e^\mu\dot\delta_2+\omega\acute e^\mu\delta_2)+G_2(e^\mu_0-v_2\acute e^\mu)\Bigr\}+\\
+Q_2\Bigl[\varphi^\mu_{2+}(+_2)-\varphi^\mu_{2-}(-_2)-\varphi^\mu_{3+}(+_2)+\varphi^\mu_{3-}(-_2)\Bigr]=0.\\
\tfrac{d}{d\tau}\Bigl\{\varphi^\mu_{1+}+\varphi^\mu_{1-}+(\varphi_{1+}-\varphi_{1-})\,e^\mu_0\Bigr\}+Q_3\Bigl[\varphi^\mu_{3+}(+)-\varphi^\mu_{3-}(-)-\varphi^\mu_{1+}+\varphi^\mu_{1-}+2\omega a_0\acute e^\mu\delta\Bigr]=0.
\end{gathered} \tag{24}$$

In the last three equations arguments (τ) for $\varphi^\mu_{1\pm}$, δ, δ_j and $(\omega\tau)$ for e^μ, $\acute e^\mu$ are omitted; we use the following notations for arguments

$$(\pm_1)\equiv(\tau\pm\sigma_1),\qquad(\pm_2)\equiv(\tau\pm\sigma_2),\qquad(\pm)\equiv(\tau\pm2\pi),$$

for the scalar products

$$\varphi^0_{j\pm} \equiv (e_0, \varphi_{j\pm}), \qquad \varphi^3_{j\pm} \equiv (e_3, \varphi_{j\pm}), \qquad \varphi_{j\pm} \equiv (e, \varphi_{j\pm}), \qquad \acute{\varphi}_{j\pm} \equiv (\acute{e}, \varphi_{j\pm}) \tag{25}$$

and

$$\begin{aligned} G_1 &= \varphi_{1+}(+_1) - \varphi_{1-}(-_1) - v_1 \breve{c}_1^{-1}\Big[\acute{\varphi}_{1+}(+_1) + \acute{\varphi}_{1-}(-_1) - 2\omega a_0 \delta_1\Big], \\ G_2 &= \breve{c}_3^{-1}\Big\{\breve{C}_2[\varphi_{2-}(-_2) - \varphi_{2+}(+_2)] + \breve{S}_2[\acute{\varphi}_{2+}(+_2) + \acute{\varphi}_{2-}(-_2)] + 2\omega v_2 a_0 \delta_2\Big\}. \end{aligned}$$

The first two equations (24) results from Eqs. (23), the third — from Eq. (6), other ones are consequence of Eqs. (2) and (3). Equations (24) are simplified with using Eqs. (12)–(17), (19) and equalities (21), resulting in the following relations for projections (25) of disturbances:

$$\varphi^0_{1\pm}(\tau) = \mp\varphi_{1\pm}(\tau), \qquad \varphi^0_{2\pm} = \mp(2v_1^2 - 1)\,\varphi_{2\pm} - 2v_1\breve{c}_1\acute{\varphi}_{2\pm}, \qquad \varphi^0_{3\pm} = \breve{S}\acute{\varphi}_{3\pm} \mp \breve{C}\varphi_{3\pm}. \tag{26}$$

The linearized system of equations (24), (26) describes evolution of small disturbances of the considered central rotational state (7), (19).

Note that scalar products of Eqs. (24) onto the vector e_3 (orthogonal to the rotational plane e_1, e_2) form the closed subsystem from 6 equations with respect to 6 functions (25) $\varphi^3_{j\pm}$:

$$\begin{gathered} \varphi^3_{j+}(+_j) + \varphi^3_{j-}(-_j) = \varphi^3_{j^*+}(+_j) + \varphi^3_{j^*-}(-_j), \\ \varphi^3_{3+}(+) + \varphi^3_{3-}(-) = \varphi^3_{1+}(\tau) + \varphi^3_{1-}(\tau), \\ \dot{\varphi}^3_{j+}(+_j) + \dot{\varphi}^3_{j-}(-_j) + Q_j\Big[\varphi^3_{j+}(+_j) - \varphi^3_{j-}(-_j) - \varphi^3_{j^*+}(+_j) + \varphi^3_{j^*-}(-_j)\Big] = 0, \\ \dot{\varphi}^3_{1+}(\tau) + \dot{\varphi}^3_{1-}(\tau) + Q_3\Big[\varphi^3_{3+}(+) - \varphi^3_{3-}(-) - \varphi^3_{1+}(\tau) + \varphi^3_{1-}(\tau)\Big] = 0. \end{gathered} \tag{27}$$

Here $j = 1, 2$, $j^* \equiv j + 1$. This system is homogeneous system with deviating arguments.

We search solutions of this system in the form of harmonics

$$\varphi^3_{j\pm} = B^3_{j\pm}\exp(-i\xi\tau). \tag{28}$$

This substitution results in the linear homogeneous system of 6 algebraic equations with respect to 6 amplitudes $B^3_{j\pm}$. The system has nontrivial solutions if and only if its determinant

$$\begin{vmatrix} E_{1+} & E_{1-} & -E_{1+} & -E_{1-} & 0 & 0 \\ 0 & 0 & E_{2+} & E_{2-} & -E_{2+} & -E_{2-} \\ -1 & -1 & 0 & 0 & E_{3+} & E_{3-} \\ (i\xi - Q_1)\,E_{1+} & (i\xi + Q_1)\,E_{1-} & Q_1E_{1+} & -Q_1E_{1-} & 0 & 0 \\ 0 & 0 & (i\xi - Q_2)\,E_{2+} & (i\xi + Q_2)\,E_{2-} & Q_2E_{2+} & -Q_2E_{2-} \\ -i\xi - Q_3 & -i\xi + Q_3 & 0 & 0 & Q_3E_{3+} & -Q_3E_{3-} \end{vmatrix} = 0$$

equals zero. Here $E_{j\pm} = \exp(\mp i\xi\sigma_j)$. This equation is reduced to the form

$$\begin{gathered} 2(\cos 2\pi\xi - 1) - \xi(Q_1^{-1} + Q_2^{-1} + Q_3^{-1})\sin 2\pi\xi + \\ + \xi^2\left(\frac{\sin^2\pi\xi}{Q_1Q_2} + \frac{\tilde{s}_3\sin\sigma_2\xi}{Q_2Q_3} + \frac{\tilde{s}_{23}\sin\sigma_1\xi}{Q_1Q_3}\right) - \frac{\xi^3\tilde{s}_3\sin\sigma_1\xi\cdot\sin\pi\xi}{Q_1Q_2Q_3} = 0, \end{gathered} \tag{29}$$

where $\tilde{s}_3 = \sin(\pi - \sigma_1)\xi$, $\tilde{s}_{23} = \sin(2\pi - \sigma_1)\xi$.

This equation describes the spectrum of frequencies ξ for transversal (with respect to the e_1, e_2 plane) small fluctuations of the string for the considered rotational state. If equation

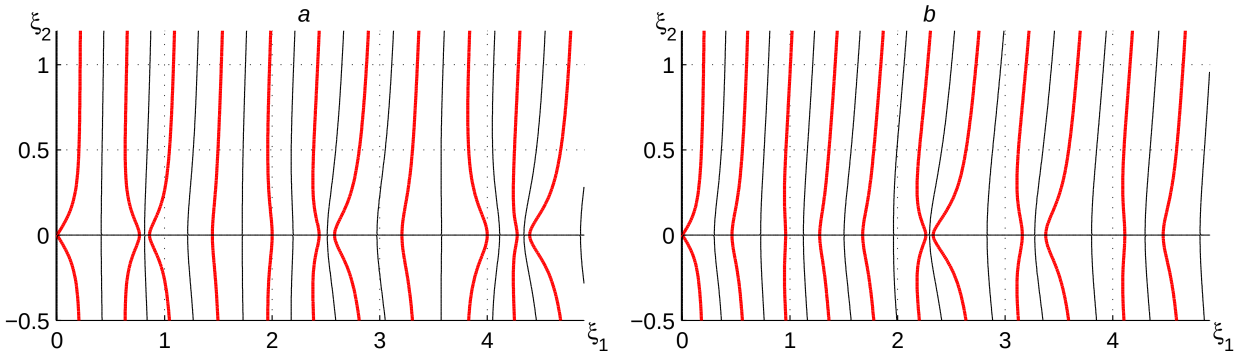

Figure 1: Zero level lines for real part (thick) and imaginary part (thin) of Eq. (29); *a)* $Q_1 = Q_2 = Q_3 = 1$, *b)* $Q_1 = 1$, $Q_2 = 0.4$, $Q_3 = 0.1$

(29) has a complex root $\xi = \xi_1 + i\xi_2$ with positive imaginary part ξ_2, the amplitude of the correspondent disturbance will grow exponentially:

$$\varphi_k^3 = B_k^3 \exp(-i\xi_1\tau) \cdot \exp(\xi_2\tau).$$

In this case the considered rotational state is unstable one [18, 21].

Analysis of the real and imaginary parts of Eq. (29) is presented in Fig. 1, where the thick and thin lines are zero level lines correspondingly for real and imaginary part of the function $f(\xi) = f(\xi_1 + i\xi_2)$ in Eq. (29) for given values Q_j. Roots of this equation are shown as cross points of a thick line with a thin line. If the values (10) Q_j are given, one can determine values ω, σ_1, v_j from Eqs. (12) – (17). For example, ω is one of the roots of the equation

$$\tan \pi\omega = \frac{2\omega(Q_1 + Q_2)}{\omega^2 - 4Q_1Q_2}, \tag{30}$$

resulting from Eqs. (14). For the state in Fig. 1*a* the root of this equation $\omega \simeq 0.766898$ is chosen. It corresponds to the mass relation $m_1 : m_2 : m_3 = 1 : 1 : 2.79$; for the state in Fig. 1*b* $\omega \simeq 2.331$, $m_1 : m_2 : m_3 = 1 : 3.12 : 13.18$. The frequency ω for the state in Fig. 1*a* is the minimal positive root of Eq. (30), corresponding to $k_1 = k_2 = 0$ in Eqs. (17) (the simplest rectilinear string segments, connecting the massive points). For the state in Fig. 1*b* the values in Eqs. (17) $k_1 = 0$, $k_2 = 1$, and the string segments are kinked lines.

The analysis of Eq. (29) for various values Q_j and ω demonstrates that all its roots are real numbers, therefore amplitudes of such fluctuations do not grow with growth of time t.

4. Disturbances in the rotational plane

One can not solve the stability problem for the cental rotational states (7) only on the base of the above studied behavior of disturbances $\varphi^3_{j\pm}$ (28) (orthogonal to the rotational plane). We are to consider small disturbances concerning to the e_1, e_2 plane. Projections (scalar products) of equations (24) onto 3 vectors e_0, $e(\tau)$, $\acute{e}(\tau)$ with using relations

$$e^\mu(\tau) = \breve{c}_1 e^\mu(\pm_1) \mp \breve{s}_1 \acute{e}^\mu(\pm_1) = \breve{C}_2 e^\mu(\pm_2) \mp \breve{S}_2 \acute{e}^\mu(\pm_2) = \breve{C} e^\mu(\pm) \mp \breve{S} \acute{e}^\mu(\pm)$$

and their analogs for $\acute{e}^\mu$ (for multiplying by vector-functions $\varphi^\mu_{j\pm}$ with different arguments) result in the system of 18 equations. Three of them are linear combinations of the other ones. The rest equations form the system of 15 differential equations with deviating arguments with

respect to 15 unknown functions of τ: $\varphi_{j\pm}$, $\acute{\varphi}_{j\pm}$ ($j = 1, 2, 3$), δ_1, δ_2, δ. The functions $\varphi^0_{j\pm}$ are excluded via Eqs. (26). Here we present 8 equations from this system, resulting from the 1-st, 3-rd and 6-th equations (24):

$$
\begin{gathered}
\varphi_{1+}(+_1) - \varphi_{1-}(-_1) + (\breve{c}_1^2 - \breve{s}_1^2)[\varphi_{2+}(+_1) - \varphi_{2-}(-_1)] - 2\breve{s}_1\breve{c}_1[\acute{\varphi}_{2+}(+_1) + \acute{\varphi}_{2-}(-_1)] = 0,\\
\breve{c}_1[\varphi_{1+}(+_1) + \varphi_{1-}(-_1) - \varphi_{2+}(+_1) - \varphi_{2-}(-_1) - 4a_0\dot{\delta}_1(\tau)] +\\
+ \breve{s}_1[\acute{\varphi}_{1-}(-_1) - \acute{\varphi}_{1+}(+_1) + \acute{\varphi}_{2+}(+_1) - \acute{\varphi}_{2-}(-_1)] = 0,\\
\breve{s}_1[\varphi_{1+}(+_1) - \varphi_{1-}(-_1) - \varphi_{2+}(+_1) + \varphi_{2-}(-_1)] +\\
+ \breve{c}_1[\acute{\varphi}_{1+}(+_1) + \acute{\varphi}_{1-}(-_1) - \acute{\varphi}_{2+}(+_1) - \acute{\varphi}_{2-}(-_1) - 4\omega a_0\delta_1(\tau)] = 0,\\
\varphi_{1+}(\tau) + \varphi_{1-}(\tau) - \breve{C}\varphi_{3+}(+) - \breve{C}\varphi_{3-}(-) + \breve{S}\acute{\varphi}_{3+}(+) - \breve{S}\acute{\varphi}_{3-}(-) = 0,\\
\acute{\varphi}_{1+}(\tau) + \acute{\varphi}_{1-}(\tau) - \breve{S}\varphi_{3+}(+) + \breve{S}\varphi_{3-}(-) - \breve{C}\acute{\varphi}_{3+}(+) - \breve{C}\acute{\varphi}_{3-}(-) = 0,\\
\varphi_{1+}(\tau) - \varphi_{1-}(\tau) - \breve{C}\varphi_{3+}(+) + \breve{C}\varphi_{3-}(-) + \breve{S}\acute{\varphi}_{3+}(+) + \breve{S}\acute{\varphi}_{3-}(-) + 2a_0\dot{\delta}(\tau) = 0,\\
\dot{\varphi}_{1+} + \dot{\varphi}_{1-} - \omega(\acute{\varphi}_{1+} + \acute{\varphi}_{1-}) = Q_3[\varphi_{1+} - \varphi_{1-} - \breve{C}(\varphi_{3+} - \varphi_{3-}) + \breve{S}(\acute{\varphi}_{3+} + \acute{\varphi}_{3-})],\\
\dot{\acute{\varphi}}_{1+} + \dot{\acute{\varphi}}_{1-} + \omega(\varphi_{1+} + \varphi_{1-}) = Q_3[\acute{\varphi}_{1+} - \acute{\varphi}_{1-} - \breve{S}(\varphi_{3+} + \varphi_{3-}) - \breve{C}(\acute{\varphi}_{3+} - \acute{\varphi}_{3-}) + 2\omega a_0\delta].
\end{gathered}
$$

In the last equations the arguments (τ) for $\varphi_{1\pm}$, $\acute{\varphi}_{1\pm}$, δ, and ($\pm$) for $\varphi_{3\pm}$, $\acute{\varphi}_{3\pm}$ are omitted.

When we search solutions of this system in the form of harmonics (28)

$$
\varphi_{j\pm} = B_{j\pm}e^{-i\xi\tau}, \qquad \acute{\varphi}_{j\pm} = \acute{B}_{j\pm}e^{-i\xi\tau}, \qquad 2a_0\delta_j = \Delta_j e^{-i\xi\tau}, \qquad 2a_0\delta = \Delta e^{-i\xi\tau}, \tag{31}
$$

we obtain the homogeneous system of 15 algebraic equations with respect to 15 amplitudes $B_{j\pm}$, $\acute{B}_{j\pm}$, Δ_1, Δ_2, Δ. The mentioned above condition of existence of nontrivial solutions for this system is vanishing the corresponding determinant.

This cumbersome determinant was calculated at the first stage in the particular symmetric case of equal masses

$$
m_1 = m_2, \qquad \sigma_1 = \frac{\pi}{2}. \tag{32}
$$

with using symbolic calculations in the package MATLAB and numerical verification.

In the case (32), resulting in equalities $\breve{s}_1 = \breve{s}_3$, $\breve{c}_1 = \breve{c}_3$, $v_1 = v_2$, $Q_1 = Q_2$, the obtained equation of the spectrum for small disturbances is equivalent to the set of equations (is factorized):

$$
\Big(\xi\tan\frac{\pi\xi}{2} - \omega\tan\frac{\pi\omega}{2}\Big)\Big(\breve{c}_1^2\xi^2 - 2\breve{s}_1\breve{c}_1\omega\xi\cot\frac{\pi\xi}{2} - Z_1\omega^2\Big) = 0, \tag{33}
$$

$$
\begin{aligned}
&\tilde{c}\,\xi(\xi^2-\omega^2)[\tilde{s}\breve{c}_1^3\xi^3 - 3\tilde{c}\breve{s}_1\breve{c}_1^2\omega\xi^2 - \tilde{s}(1+3\breve{s}_1^2)\,\breve{c}_1\omega^2\xi + \tilde{c}\breve{s}_1 Z_1\omega^3] = (\tilde{c}_1\breve{c}_1^2\xi^2 + 2\tilde{s}_1\breve{s}_1\breve{c}_1\omega\xi - \tilde{c}_1 Z_1\omega^2)\times\\
&\times 4Q_3\Big[\breve{c}_1\xi^3\cos\frac{3\pi\xi}{2} + 2(\tilde{c}\tilde{s}_1\breve{s}_1\omega + \tilde{s}\tilde{c}_1\breve{c}_1Q_3)\,\xi^2 + (\tilde{c}\tilde{c}_1\breve{c}_1\omega + 2\tilde{s}\tilde{s}_1\breve{s}_1Q_3)\,\omega\xi + \tilde{c}\tilde{s}_1\breve{s}_1\omega^3\Big].
\end{aligned} \tag{34}
$$

Here

$$
Z_1 = 1 + \breve{s}_1^2, \quad \tilde{c} = \cos\pi\xi, \quad \tilde{s} = \sin\pi\xi, \quad \tilde{c}_1 = \cos\sigma_1\xi, \quad \tilde{s}_1 = \sin\sigma_1\xi.
$$

Analysis of equation (33) (decomposing into two factors) for complex $\xi = \xi_1 + i\xi_2$ shows, that for all values ω and $Q_1 = \frac{1}{2}\omega\breve{s}_1/\breve{c}_1$ all roots of this equation are real numbers and form a countable set. Their behavior is similar to that for roots of Eq. (29).

But roots of equation (34) have other properties. These roots are shown in Fig. 2 as cross points of thick and thin lines for two types of rotational states: for $Q_1 = Q_2 = 1/4$, $\omega = 1/2$, $v_1 = v_2 \simeq 0.707$ on the left and $Q_1 = Q_2 = 1$, $\omega \simeq 0.766898$, $v_j \simeq 0.934$ on the right for various values Q_3 and corresponding $m_3 = \gamma a_0/Q_3$ (15).

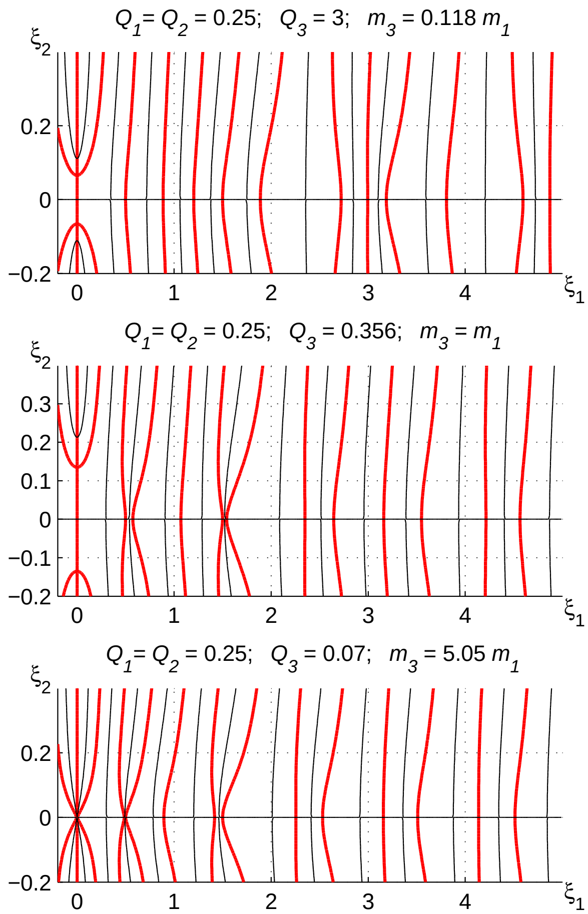

Figure 2: Zero level lines for real part (thick) and imaginary part (thin) of Eq. (34) for specified values Q_j and m_3

Fig. 2 demonstrates, that Eq. (34) has complex roots $\xi = \xi_1 + i\xi_2$ with positive imaginary parts $\xi_2 > 0$, if the value Q_3 (and the corresponding mass m_3) is not too small or too large. These roots generate exponentially growing modes of disturbances: $|\varphi| \sim \exp(\xi_2\tau)$.

The same picture takes place for other values ω. So we may conclude, that the cental rotational states (7) of the type (32) are *unstable* with respect to small disturbances, if the central mass is restricted by

$$0 < m_3 < m_{3cr}. \tag{35}$$

The critical value m_{3cr} (and the corresponding value Q_{3cr}) is determined from the condition of vanishing all complex roots of Eq. (34) (with $\operatorname{Im}\xi > 0$) for $m_3 > m_{3cr}$. Thus, we obtain the threshold effect in stability properties: if $m_3 \geq m_{3cr}$, all roots of Eq. (34) are real ones and the state is stable. But in the case $m_3 < m_{3cr}$ the state is unstable.

For any values $Q_1 = Q_2$ and any (arbitrarily small) value m_3 from the interval (35) there are exists the pure imaginary root $\xi^* = i\xi_2^*$ ($\xi_2^* > 0$). It tends to 0 at $m_3 \to 0$. For the states with $\omega \leq 0.5$, $Q_1 \leq 0.25$ equation (34) has no other complex roots except $\xi^* = i\xi_2^*$. If mass m_3 increases, the value ξ_2^* (increment of disturbances' growth) increases too, reaching the maximal value for masses m_3, close to m_1 in order of magnitude. If m_3 increases further (Q_3 diminishes) the increment ξ_2^* decreases and vanishes at the critical value $m_{3cr} \equiv m_{3cr}^* = \gamma a_0/Q_{3cr}^*$, in the (35). For the case $Q_1 = Q_2 = 1/4$ the value $m_{3cr}^* \simeq 5.05\, m_1$.

The critical value Q_{3cr}^*, corresponding to vanishing the root ξ^*, may be calculated, if we

substitute $\xi = i\xi_2$ into Eq. (34) and analyze its behavior at $\xi_2 \to 0$:

$$\breve{s}_1 Z_1 \omega^5 \xi_2 - 2Q_3 Z_1 \omega^4 \frac{\breve{c}_1(1+\cosh\pi\xi_2) + \breve{s}_1\omega\sinh\pi\xi_2}{\cosh\pi\xi_2} + \phi(\xi_2) = 0, \qquad \phi(\xi_2) = \mathcal{O}(\xi_2^3).$$

The function $\phi(\xi_2) = \phi_3\xi_2^3 + \phi_5\xi_2^5 + \ldots$ is positive for $\xi_2 > 0$ (contains only positive summands), so the root $\xi_2 = \xi_2^*$ of this equation exists only under the condition $2Q_3(2\breve{c}_1 + \pi\breve{s}_1\omega) > \breve{s}_1\omega$. From this condition and equalities (14) and (32) one can find the critical value $Q_{3cr} = Q_{3cr}^*$ for the root $i\xi_2^*$:

$$Q_{3cr}^* = \frac{1}{2\pi + 4\breve{c}_1(\omega\breve{s}_1)^{-1}} = \frac{1}{2\pi + 2Q_1^{-1}}. \tag{36}$$

In the limit $m_1 \to 0$, ($Q_1 \to \infty$) expression (36) takes the form, coinciding with the critical value in the condition of instability $Q > Q_{cr}^* = (2\pi)^{-1}$ [**?**] for the cental rotational states of the closed string with $n = 1$ massive point. The equation of the small disturbances spectrum for this string [**?**]

$$\xi^3 + 2Q(2\xi^2 + \omega^2)\tan\pi\xi + 4Q^2\xi\tan^2\pi\xi - \omega^2\xi = 0$$

results from Eq. (34) ($Q \equiv Q_3$) in the mentioned limit $m_j \to 0$, $j = 1, 2$.

For values $\omega > 1/2$ the structure of complex roots of Eq. (34) is more complicated. Because of the symmetry we shall count only roots in the quadrant $\xi_1 \geq 0$, $\xi_2 > 0$. For $Q_1 > 1/4$ in certain interval of values m_3 the second complex root of Eq. (34) exists (denoted below by $\xi^\diamond$); for larger values Q_1 corresponding to $v_1 \to 1$ the third and other roots appear. In particular, in Fig. 2 for $Q_1 = 1$ the root $\xi^\diamond$ appears, if $Q_3 < 6.667$; for $Q_3 = 1$ the complex roots are: $\xi^* \simeq 0.262\,i$; $\xi^\diamond \simeq 3.64 + 0.137\,i$; the third root $\xi^\triangle \simeq 5.49 + 0.017\,i$ (it exists in the narrow interval $0.86 < Q_3 < 1.65$).

For different values Q_1 the critical value Q_{3cr} is determined by vanishing the root ξ^* or $\xi^\diamond$. For the states in Fig. 2 for $Q_1 = 1/4$ the first variant takes place and $Q_{3cr} = Q_{3cr}^* \simeq 0.07$; for $Q_1 = 1$ we see the second variant: $Q_{3cr} = Q_{3cr}^\diamond \simeq 0.05$.

In the case $m_3 = 0$, corresponding to $Q_3 \to \infty$, there is no massive point at the center, and we have the linear rotational state with $n = 2$. In this case equation (34) takes the form

$$\Big(\xi + \omega\tan\frac{\pi\omega}{2}\tan\frac{\pi\xi}{2}\Big)\xi\sin\pi\xi = 0. \tag{37}$$

For all values ω it has only real roots. So the linear rotational state with $n = 2$ of the type (32) are stable.

Stability also takes place for the case $Q_3 = 0$ ($m_3 \to \infty$).

Generalization of the above analysis for the case of arbitrary masses, in particular, $m_1 \neq m_2$ results in rather complicated equation of the small disturbances spectrum. In the particular case $Q_3 = 0$ ($m_3 \to \infty$) this equation have the form

$$\begin{aligned}
\Big\{&\tilde{s}\tilde{s}_1\tilde{s}_3\breve{c}_1^3\breve{c}_3^3\xi^6 - 3\tilde{s}_1(\tilde{s}\tilde{c}_3 + \tilde{c}\tilde{s}_3)\breve{s}_3\breve{c}_1^3\breve{c}_3^2\omega\xi^5 + \tilde{s}_1\Big[\tilde{c}\tilde{c}_3\breve{s}_3(4\breve{c}_1\breve{s}_3 + 9\breve{s}_1\breve{c}_3) - 2\tilde{s}\tilde{s}_3\breve{c}_1 Z_3\Big]\breve{c}_1^2\breve{c}_3\omega^2\xi^4 + \\
&+\Big[\tilde{s}\tilde{s}_1\tilde{c}_3\breve{s}_3(6\breve{c}_1\breve{c}_3\breve{s}_1\breve{s}_3 + \breve{c}_1^2 Z_3 + 3\breve{c}_3^2 Z_1) + \tilde{c}\Big(\tilde{s}_1\tilde{s}_3 Z_3(\breve{c}_1\breve{s}_3 + 3\breve{s}_1\breve{c}_3) - 6\tilde{c}_1\tilde{c}_3\breve{c}_3\breve{s}_1\breve{s}_3^2\Big)\breve{c}_1\Big]\breve{c}_1\omega^3\xi^3 - \\
&-\Big[\tilde{c}(3\tilde{s}\breve{c}_1\breve{s}_3 + 4\tilde{c}_1\tilde{s}_3\breve{s}_1\breve{c}_3)\,Z_3\breve{s}_1 + 2\tilde{s}_1\tilde{c}_3(4\tilde{c}_1\tilde{c}_3\breve{s}_1^2\breve{s}_3^2 - \tilde{s}_1\tilde{s}_3 Z_1 Z_3)\,\breve{c}_3\Big]\breve{c}_1\omega^4\xi^2 + \\
&+\Big[\tilde{c}(2\tilde{c}_1\tilde{c}_3\breve{s}_1^2 - \tilde{s}_1\tilde{s}_3 Z_1)\,Z_3\breve{c}_1\breve{s}_3 - \tilde{s}\tilde{c}_1\tilde{s}_3\breve{s}_1 Z_3(2\breve{c}_1\breve{s}_1\breve{s}_3 + \breve{c}_3 Z_1)\Big]\omega^5\xi + \tilde{c}\tilde{s}_1\tilde{c}_3\breve{s}_1\breve{s}_3 Z_1 Z_3\omega^6 + \\
&+(1 \leftrightarrow 3)\Big\}\xi(\xi^2 - \omega^2) = 0.
\end{aligned} \tag{38}$$

Here $Z_3 = 1 + \breve{s}_3^2$, $\tilde{c}_3 = \cos(\pi - \sigma_1)\,\xi$, $\tilde{s}_3 = \sin(\pi - \sigma_1)\,\xi$, the symbol $(1 \leftrightarrow 3)$ means repeating the same terms with transposed indices "1" and "3".

Equation (38) has only real roots, so the rotational states (7) with the infinitely heavy mass $m_3 \to \infty$ at the center are stable.

Linear rotational states with $n = 2$ ($m_3 = 0$) are also stable for any values ω, m_1, m_2.

But for intermediate values m_3 from the interval (35) the spectrum of small disturbances

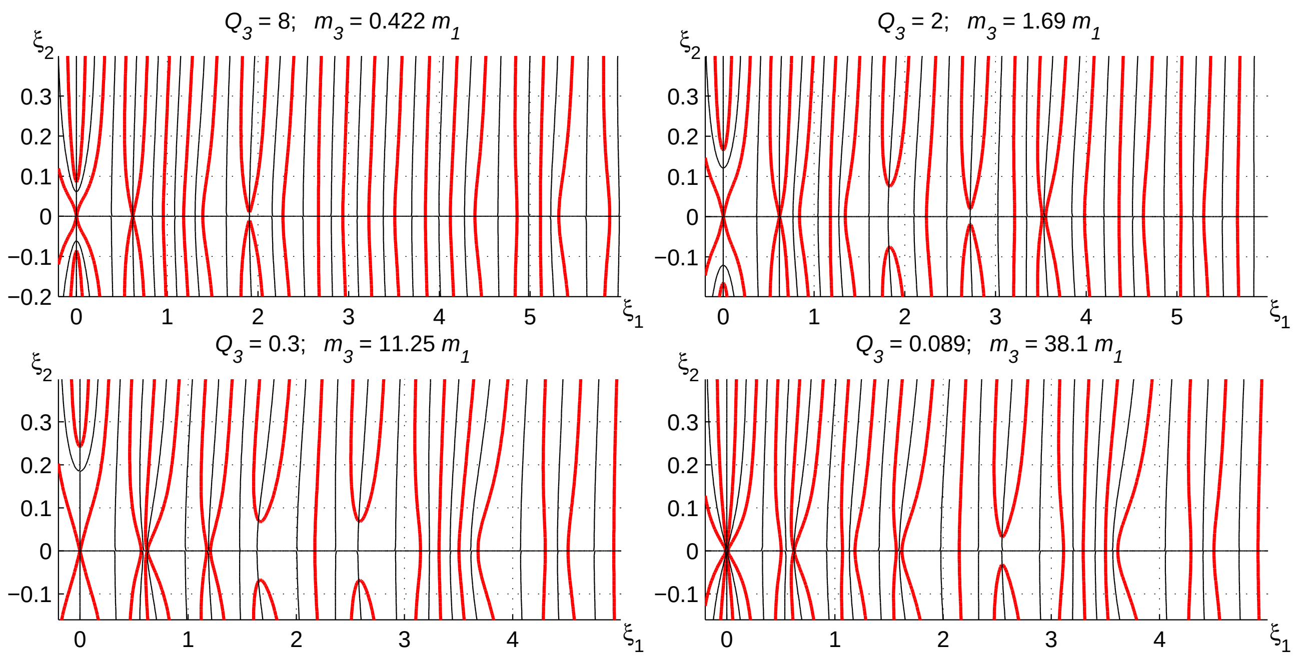

Figure 3: Frequencies of small disturbances for the state with $Q_1 = 1$, $Q_2 = \frac{1}{4}$ ($m_2 \simeq 10.5\,m_1$) and specified Q_3, m_3

For this state $\omega \simeq 0.62025$. Behavior of complex roots is similar to the case (32): there is the range (35) $0 < m_3 < m_{3cr}$ of existence of complex roots, hence the rotational state is unstable in this range.

There are three types of complex roots in Fig. 3 (similarly to Fig. 2): the imaginary root ξ^*, the complex roots $\xi^\diamond$ and $\xi^\triangle$. If we suppose $m_1 = 1$ (it is equivalent to $\gamma a_0 \simeq 3.376$), that the range of existence ξ^* is $0 < m_3 < 38.1$, for $\xi^\diamond$ it is $0.42 < m_3 < 30.7$, and for $\xi^\triangle$ this range is $1.64 < m_3 < m_{3cr} \simeq 67.5$. For other parameters of the state the critical mass m_{3cr} is determined from vanishing the root ξ^* or other roots.

Generalization of the expression (36) for the critical value Q_{3cr}^*, determined from vanishing the root ξ^* takes the form

$$(Q_{3cr}^*)^{-1} = 2\pi + Q_1^{-1} + Q_2^{-1}.$$

Taking into account Eq. (15) $m_3 = \gamma a_0 / Q_3$ we obtain the critical value of the central mass

$$m_{3cr}^* = 2\pi\gamma a_0 + \frac{m_1}{\sqrt{1 - v_1^2}} + \frac{m_2}{\sqrt{1 - v_2^2}} \equiv E - m_3. \tag{39}$$

It coincides with energy of this state of the string without contribution of the mass m_3 [7, 10].

We may conclude, the central rotational state is unstable if the central mass m_3 is nonzero and less than energy of the string with other massive points.

5. Numerical experiments

In this section the above results connected with stability or instability of rotational states are verified in numerical experiments. Slightly disturbed rotational states (7) of the closed string with $n = 3$ masses are simulated numerically in comparison with the similar states of the linear string baryon model *q-q-q* [3]. We use the approach, suggested in Refs. [18, 20, 21]. It includes solving the initial-boundary value problem, by other words, calculation of classical motion of the system on the base of given initial position of the string $X^\mu|_{ini} = \rho^\mu(\tilde\sigma)$ and initial velocities of its points $\dot X^\mu|_{ini} = v^\mu(\tilde\sigma)$, $\tilde\sigma = \tilde\sigma(\sigma)$.

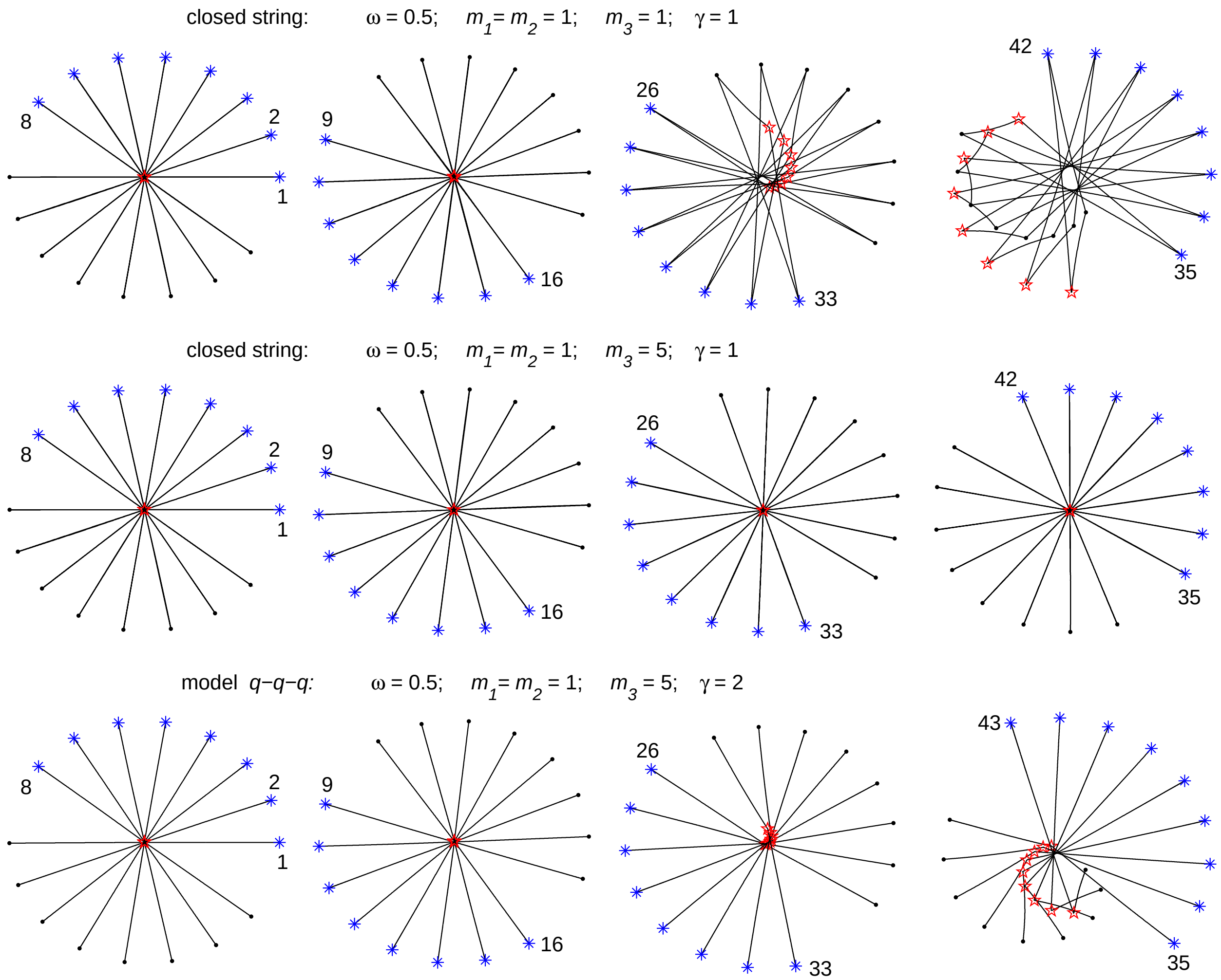

Figure 4: Numerical simulation of disturbed rotational motions

If this initial conditions correspond to a central rotational state (7) with a small disturbance, for example $\rho^\mu(\tilde\sigma) = \rho^\mu_{rot}(\tilde\sigma) + \delta\rho^\mu(\tilde\sigma)$, $v^\mu(\tilde\sigma) = v^\mu_{rot}(\tilde\sigma) + \delta v^\mu(\tilde\sigma)$, we calculate slightly disturbed rotational motion of the system. In Fig. 5 the examples of such a motion for the closed string and the linear model *q-q-q* are represented as a set of "photographs" or positions of the string in e_1, e_2-plane. These positions (sections t = const of the world surface) are numbered in order of increasing t with spacing in time $\Delta t = 0.25$; the numbers are shown near the first massive point marked by the asterisk. The second and the third (central) masses are marked by the point and the pentagram correspondingly. Parameters of the states are pointed out, they correspond to $Q_1 = Q_2 = \frac{1}{4}$ (compare with Fig. 2).

In three cases in Fig. 4 the small disturbance in initial data is in the form $\delta\rho^\mu = 0$ and $\delta v^\mu(\tilde\sigma) = 0.01\sin(\tilde\sigma - \tilde\sigma_2)$ in the interval $\tilde\sigma_2 < \tilde\sigma < \tilde\sigma_3$ ($\tilde\sigma_3 - \tilde\sigma_2 = \pi$) between the points m_2 and m_3, but δv^μ equals zero on all other points.

As one can see in Fig. 4, if the value m_3 is in the interval (35), and the increment ξ_2^* of disturbances' growth is large enough (the case with $m_3 = 1$), the evolution of growing disturbances results in going away the central mass. The string changes into rotating curvilinear triangle and massive points change their positions.

In the case $m_3 \ge m_{3cr}$ the motion is stable and the massive point m_3 remains near the rotational center. Similar picture takes place, if $m_3 \simeq m_{3cr}$, in particular $m_3 = 5$ is close to $m_{3cr} \simeq 5.05$. In this case the increment ξ_2^* is very small and the disturbed motion looks like a stable one.

It is interesting to compare this picture with slightly disturbed rotational states for the linear string baryon model q-q-q. If tension γ is twice large than for the closed string and masses m_j are the same, parameters of a central rotational state ω, Q_j and v_j coincide for both models. It was shown in Ref. [**?**] that the spectrum of small disturbances (20) for these states of the system q-q-q is described by the equation

$$\frac{m_3(1-v_i^2)\,\xi(\xi^2-\omega^2)}{m_1v_1\omega(\xi^2+\omega^2)} = \frac{\breve c_1(Q_1^2\kappa_1-\xi^2)-2\breve s_1Q_1\xi}{\breve s_1(Q_1^2\kappa_1-\xi^2)+2\breve c_1Q_1\xi} + \frac{\breve c_3(Q_2^2\kappa_2-\xi^2)-2\breve s_3Q_2\xi}{\breve s_3(Q_2^2\kappa_2-\xi^2)+2\breve c_3Q_2\xi},$$

where $\kappa_j = 1 + v_j^{-2}$. For any value $m_3 > 0$ the imaginary root $\xi = i\xi_2^*$ of this equation exists (there is no critical maximal value), so the picture of stability differs from that for the closed string: central rotational states of the linear string baryon model q-q-q are unstable for any masses $m_3 > 0$.

6. Regge trajectories for unstable states

Rotational states of the closed string with n massive points are applied for describing orbitally excited baryons [4, 8] and the Pomeron trajectory [10, 11], corresponding to possible glueball states.

For linear and central rotational states (7) the energy E and angular momentum J are used in Refs. [4, 10, 11] in the following form:

$$E = 2\pi\gamma a_0 + \sum_{j=1}^n \frac{m_j}{\sqrt{1-v_j^2}} + \Delta E_{SL}, \tag{40}$$

$$J = L + S = \frac{\gamma a_0^2}{2\omega}\Big(2\pi + \sum_{j=1}^n \frac{v_j^2}{Q_j}\Big) + \sum_{j=1}^n s_j. \tag{41}$$

Here s_j are spin projections of massive points (quarks or valent gluons), ΔE_{SL} is the spin-orbit contribution to the energy in the following form [4]:

$$\Delta E_{SL} = \sum_{j=1}^n [1-(1-v_j^2)^{1/2}](\Omega\cdot\mathbf{s}_j).$$

If the string tension γ, values m_j and the type of rotational state are fixed, we obtain the one-parameter set of motions with different values E and J. The parameter of this set is any of the values: ω, a_0, E, J; other values are expressed via relations (12)–(17). These states lay at quasilinear Regge trajectories. If a central (or linear) rotational state (7) is the simplest

one, that is $k_1 = k_2 = 0$ in Eq. (17) the asymptotic behavior of the corresponding Regge trajectory in the limit $E \to \infty$ is [10, 11]:

$$J \simeq \alpha'(E - m_3)^2, \qquad \alpha' = \frac{1}{4\pi\gamma}. \tag{42}$$

Below we apply the central rotational states to describing the Pomeron trajectory [11, 23], corresponding to glueball states. We suppose that the value S in Eq. (41) corresponds to the maximal total momentum (41), that is $S = 2$ for 2-gluon glueballs and $S = 3$ for 3-gluon glueballs [15, 16]. Other values of model parameters are [10, 11]:

$$\gamma = 0.175 \text{ GeV}^2, \quad m_1 = m_2 = m_3 = 750 \text{ MeV}. \tag{43}$$

This tension γ ... 0.9 GeV^{-2}. Es-
timations of gl ... tice calculations
[24] yield valu

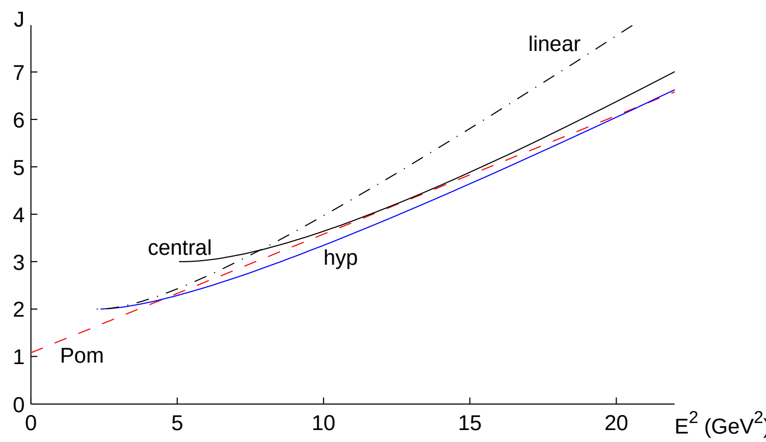

Figure 5: Regge trajectories for rotational states

Regge trajectories or graphs $J = J(E^2)$ of expressions (40) and (41) for linear, central and hypocycloidal rotational states with parameters (43) are presented in Fig. 5. These trajectories lie close to the pomeron trajectory [23, 25]

$$J \simeq 1.08 + 0.25E^2; \tag{44}$$

it is shown as the dashed line. The dash-dotted line corresponds to the linear state with $n = 2$, the hypocycloidal state is “triangle” type state with $n = 2$ (the third vertex of the triangle is the massless point), considered in Refs. [10, 11]. For the central rotational state $n = 3$.

These Regge trajectories are nonlinear for small E and tend to linear if $E \to \infty$. Their slope α' in this limit tends to the limit (42) for linear and central rotational states and to the value $\alpha' = \frac{3}{8}(2\pi\gamma)^{-1}$ for “triangle” hypocycloidal states [11].

We have shown in Sect. 4 that the central rotational states (7) with equal masses (43) are unstable for all energies on the classic level. But this does not mean disappearance or terminating corresponding Regge trajectories in Fig. 5. The straight consequence of this instability is the contribution to width of a hadron state.

String models describe only excited hadron states with large orbital momenta L. These states are unstable with respect to strong interactions and have rather large width Γ. In string interpretation of excited hadron this width is connected with probability of string breaking;

this probability is proportional to the string length ℓ [26, 27]. The value ℓ is proportional to the string contribution E_{str} to energy E of a hadron state, in particular, for rotational states (7) this contribution to the expression (40) is $E_{str} = 2\pi\gamma a_0$.

Therefore, the component of width Γ_{br}, connected with string breaking, is proportional to E_{str} with the factor 0.1 resulting from particle data [27, 28]:

$$\Gamma_{br} \simeq 0.1 \cdot E_{str} = 0.2 \cdot \pi\gamma a_0. \tag{45}$$

The contribution Γ_{inst} to width Γ due to instability of string central rotational states (7) with respect to small disturbances is determined from the increment ξ_2 of exponential growth

$$|\varphi| \sim \exp(\xi_2\tau) = \exp(\xi_2 a_0^{-1} t).$$

For the central state in Fig. 5 with parameters (43) the increment $\xi_2 = \xi_2^*$, so its width is

$$\Gamma_{inst} \simeq \frac{\xi_2^*}{a_0}. \tag{46}$$

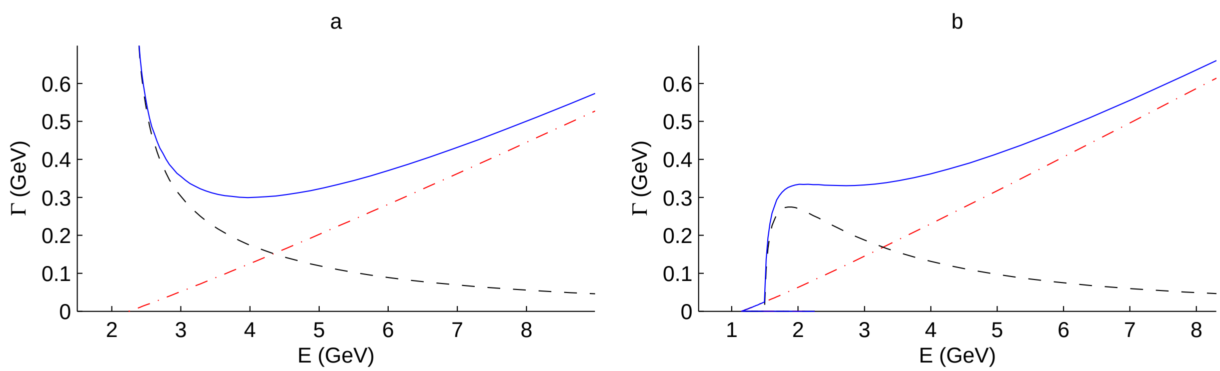

Figure 6: Width $\Gamma(E)$ (47) (solid line) as the sum of Γ_{br} (45) (dash-dotted line) and Γ_{inst} (46) (dashed line) for central states (a) with parameters (43); (b) with $m_1 = m_2 = 200$ MeV

In Fig. 6a we see that the central state with equal masses (43) is unstable for all energies E. The corresponding width component $\Gamma_{inst}(E)$ (46) tends to zero at large E (because a_0 increases) and tends to infinity in the limit $E \to E_{min} = \sum m_j$. In this limit both values $\xi_2^* \to 0$, $a_0 \to 0$ but their ratio $\Gamma_{inst} \to \infty$. So in the limit $E \to E_{min}$ the central states with $m_3 < m_1 + m_2$ are unstable (note that string models are not applicable in this limit).

Behavior of width $\Gamma(E)$ (47) for central rotational states of the system with $m_3 > m_1 + m_2$ is presented in Fig. 6b with the same notations. Here m_3 and γ are the same values (43). In this case the threshold effect (39) of instability exists, so the "instability" width $\Gamma_{inst}(E)$ equals zero for energies $E < E_{cr} = 2m_3$ (here it is 1.5 GeV). For $E > E_{cr}$ the value $\Gamma_{inst}(E)$ exceeds $\Gamma_{br}(E)$ in the certain interval, but if E increases, $\Gamma_{inst}(E)$ tends to zero and $\Gamma_{br}(E)$ increases linearly.

Conclusion

For linear rotational states (7) of a closed string with 2 massive points and central rotational states (with the mass m_3 at the rotational center) the stability problem is solved on the classic level. It is shown that the linear states are stable with respect to small disturbances in linear approximation, but the central states are unstable, if the central mass it less than the critical value (39). This value equals energy of the string with other massive points.

This threshold effect was investigated both analytically (instability is connected with exponentially growing modes in the spectrum of small disturbances of a rotational state) and in numerical experiments.

Instability of these central states results in some manifestations, in particular, in additional width of excited hadron states. It is shown that this contribution makes essential changes in linear dependence $\Gamma \sim E$.

Acknowledgment

The author is grateful to Russian foundation of basic research for the support (grant 05-02-16722).

References

[1] A. Chodos and C. B. Thorn, Nucl. Phys. **B72**, 509 (1974).

[2] X. Artru, Nucl. Phys. **B85**, 442 (1975).

[3] V. P. Petrov and G. S. Sharov, Mathem. Modelirovanie, **11**, 39 (1999), hep-ph/9812527.

[4] G. S. Sharov, Phys. Atom. Nucl. **62**, 1705 (1999).

[5] I. Yu. Kobzarev, B. V. Martemyanov, and M. G. Shchepkin, Sov. Phys. Usp. **35**, 257 (1992).

[6] M. S. Plyushchay, G. P. Pronko, and A. V. Razumov. Theor. Math. Phys. **63**, 389 (1985).

[7] G. S. Sharov, Theor. Math. Phys. **113**, 1263 (1997); Phys. Rev. **D58**, 114009 (1998), hep-th/9808099.

[8] A. Inopin, G. S. Sharov, Phys. Rev. **D63**, 054023 (2001), hep-ph/9905499.

[9] L. D. Soloviev, Phys. Rev. **D58**, 035005 (1998); **D61**, 015009 (2000), hep-ph/9907486.

[10] G. S. Sharov, arXiv 0712.4052.

[11] G. S. Sharov, Phys. Atom. Nucl. **71**, 574 (2008).

[12] L. D. Soloviev, Theor. Math. Phys. **126**, 203 (2001), hep-ph/0006010; S. V. Talalov, hep-ph/0101028; F. J. Llanes-Estrada, S. R. Cotanch, P. Bicudo, *et al.,* Nucl. Phys. A **710**, 45 (2002).

[13] A. P. Szczepaniak and E. S. Swanson, Phys. Lett. B **577**, 61 (2003); hep-ph/0308268.

[14] J.M. Pons, J.G. Russo, and P. Talavera, Nucl. Phys. B **700**, 71 (2004); hep-th/0406266.

[15] E. Abreu and P . Bicudo, hep-ph/0508281.

[16] V. Mathieu, C. Semay, and B. Silvestre-Brac, Phys. Rev. D **74**, 054002 (2006); hep-ph/0605205.

[17] M. I. Krivoruchenko and B. V. Martemyanov, Phys. Rev. D **71**, 017502 (2005), hep-ph/0502021.

[18] G. S. Sharov, Phys. Rev. **D62**, 094015 (2000), hep-ph/0004003.

[19] G. S. Sharov, Theor. Math. Phys. **140**, 242 (2004).

[20] G. S. Sharov, Izv. vuzov, Fizika **45**, 65 (2002), hep-ph/0110416.

[21] G. S. Sharov, Phys. Atom. Nucl., **65**, 906 (2002).

[22] A. E. Milovidov and G. S. Sharov, Theor. Math. Phys. **142**, 61 (2005); hep-th/0512330; G. S. Sharov, hep-th/0603252.

[23] S. Donnachie, G. Dosch, O. Nachtmann and P. Landshoff, *Pomeron Physics and QCD*, Cambridge University Press, 2002.

[24] C. W. Bernard, Phys. Lett. B **108**, 431 (1982); F. D. R. Bonnet, P. O. Bowman, D. B. Leinweber and A. G. Williams, Phys. Rev. D **62**, 051501 (2000), hep-lat/0002020; P. J. Silva and O. Oliveira, Nucl. Phys. B **690**, 177 (2004), hep-lat/0403026.

[25] H. B. Meyer and M. Teper, Phys. Lett. B **605**, 344 (2005); H. B. Meyer, hep-lat/0508002.

[26] I. Yu. Kobzarev, B. V. Martemyanov, and M. G. Shchepkin, Sov. J. Nucl. Phys. **48**, 344 (1988).

[27] K. S. Gupta and C. Rosenzweig. Phys. Rev. D **50**, 3368 (1994), hep-ph/9402263.

[28] E. Klempt and A. Zaitsev, arXiv:0708.4016